Perceptual Organization with active contour functions : application to aerial and medical images

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Abstract. In this paper, we describe a new method of perceptual organization of continuation, applied to the extraction of thin networks on medical and satellite images. The key point of our approach is to consider perceptual organization as a problem of optimization. We first qualify the quality of a grouping with a class of functions inspired by the energy functions used for active contours optimization (involving curvature, co-circularity, grey levels, and orientation). We show how such functions can be computed recursively, and we propose an optimization algorithm from a local to a global level (related to dynamic programming). This is followed by the selection procedure which rates and extracts principal groupings. We show the validity of our approach with synthetic images and natural data such as aerial and medical images.

Keywords: Perceptual Organization, Dynamic Programming, Non convex Objects Extraction, Segmentation.

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Summary

What is the original contribution of this work ?

In this paper, we describe a new method for Perceptual Grouping of continuation, able to deal with non convex objects. We define a new class of quality functions derived from active contours energy functions. We propose a new iterative scheme for optimizing this quality functions from a local to a global level. Then we describe an algorithm which extracts principal groupings automatically. Finally we show how our method gives a solution to a difficult problem of optimization.

Why should this contribution be considered important ?

We combine two existing models, active contours and models of optimization from a local to global level, in order to obtain a clear formalism for Perceptual Grouping of continuation. We present results on synthetic images and real data: medical and satellite images. The high quality of the results we obtained with average parameters on various classes of shapes clearly show the stability of our algorithm and the validity of our approach.

What is the most closely related work by others and how does this work differ ?

The most closely related works are in the area of Perceptual Organization with a locally connected network of elements. This paper gives improvements to the local to global optimization scheme proposed by A. Sha'ashua, S. Ullman [12], using a different algorithm and Snakes-like functions such as defined by M. Kass, A. Witkins, D. Terzopoulos [5] and M. O. Berger [1].

How can other researchers make use of the results of this work ?

Our method can be used to structure noisy and uncomplete images provided by low level processing (such as edges or crest-lines detection). Our method is an intermediate stage between Segmentation and Interpretation levels. It can be used for example to establish control loops between low level processings at different scales and perceptual grouping. Other applications can range from the extraction of unknown shapes in very noisy images, to the initialization of snakes or of a scene interpretation process.

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1 Introduction

The detection of thin networks is important for many problems of image analysis, as it is the case with the detection of roads in satellite images or blood vessels in medical images. For this class of problems, the lines detected are discontinuous and some of them correspond to no object in the image, even with very careful low level extraction [8].

This is due to several causes: noise, texture, junctions and also different structures in the images that do not correspond to roads or blood vessels. We propose in this paper a Perceptual Grouping method for the extraction of thin networks as those encountered in satellite or medical images.

One of the ways to represent Perceptual Grouping is extracting organized structures from visual data and grouping together primitives coming from common causes. These groupings are the result of an organization scheme following basic relations between visual primitives such as continuation, symmetry, similarity or object-background separation. Gestalt Psychology [14], or psychology of shape, used experimental observations of these grouping phenomena to show how human vision organizes the representation of the world instead of seing it in a chaotic way.

Previous work regarding pixel organization, which is the topic of this paper, can be divided in two different techniques: algorithmic techniques [4],[6], [9], [13] and optimization techniques [3] [11] The method described in this paper falls within the scope of optimization techniques. First we define "Snakes like" functions qualifying the quality of a grouping. We then present a method to optimize these functions over the whole image and select automatically the best groupings. The method is based on a two-stage algorithm which first aggregates the image elements in regard to the quality function (involving curvature, co-circularity, grey levels, and orientation) and then selects the best groupings. This quality function is optimized using a procedure from a local to a global level related to dynamic programming. The optimal groupings are then selected according to their global quality and give a new structuration of image primitives, based on smoothed continuation. Our method is able to recognize large linear structures in images. These structures may be discontinous and embedded with noise. We show how it is possible to extract most of the networks with no prior high level knowledge of the scene but using only very general geometric relations such as curvature and co-circularity.

Results are obtained on synthetic, satellite and medical scenes, using standard parameters. They clearly show the applicability of our approach and the interest of our method for initiating a scene interpretation process.

This paper is divided into five parts. Previous work in the field of Perceptual Grouping is presented in section 2. Section 3 defines Regularization and Image Quality functions and shows how the computation of these quality functions has been made recursive. The optimization algorithm and the algorithm of selection of the best curves are presented in section 4. Details about the different parameters and the implementation are given in section 5. We conclude by presenting and discussing results obtained on synthetic and natural images in section 6.

2 Relationship to prior works

In this section we present a short overview of the work in Perceptual Organization related to curve detection in noisy images. This complex combinatorial problem is mostly expressed as grouping together pixels or interests points into larger structures (such as chains or regions). Other approaches are possible, according to the nature and complexity of the primitives used for grouping. These primitives can be grouped according to undetermined shapes (such as segments, curves, or regions) or well defined (circles, squares, ellipses). Usually, these groupings are used to more easily initiate a model-based shape recognition system. Very few attempts have been made to group larger structures.

The use of Perceptual Grouping techniques in computer vision is not a recent idea. In 1976, Marr [7] suggested the idea of a "primal sketch" to use not only information from contour segmentations but also groupings of elements from the image according to primitives such as curves or lines. This idea was never implemented and Perceptual Grouping was not really used in Computer Vision before Lowe's works .

According to Lowe [6], each grouping has its own statistic feature, representing the probability to be an accidental grouping, thus qualifying its saliency. When the probability of a certain grouping is low, it becomes salient for the scene. The most significant relationships between primitives for a grouping are those who remain invariant for various points of view : co-linearity, symmetry, proximity, etc. His works showed how Perceptual Grouping could be used to efficiently structure images produced by less accurate or biased low-level processings.

Parent and Zucker [15], [10] proposed quality functions for perceptual organization using the relation of continuity. Their main contribution is in the definition of functions taking into account the saliency of curves as the human eye would do. These functions use co-circularity, curvature, derivative of curvature, grey levels, etc. Perceptual organization is accomplished here by a global approach of combinatorial optimization. Herault[3] uses a quality function based on co-circularity defined by Parent and Zucker for the qualification of the saliency of curves. Co-circularity is calculated with contour information and the orientation of edge points. The approach here is essentially global and the optimization is achieved by the methods of annealing and mean field annealing.

Sha'ashua [11] presents an interesting iterative scheme allowing a local to global optimization for some classes of functions. Unfortunately, his work does not provide good results on real images, since it is extremely sensitive to noise and doesn't take into account global aspects of curves.

Models of active contours or Snakes [5] have been developed for the detection of structures containing weak edges, such as those found in medical images. According to these models, the trace of a contour is changed iteratively to minimize an energy function. The energy function is divided into independant and yet opposed internal and external influences, which can be controlled easily. This model, however, illustrates an important weakness of the approach. As the energy function is not convex, there is no direct way to find its minimum. Iterative methods (such as Gradual Non Convexity Algorithm [1]) are available, but they require an initialization close to an optimal solution. Finding this initialization automatically is not easy for most applications, particularly when different objects are present in a scene. These models give an attracting formalism for the definition of quality functions of smoothness and continuity, but in return, give a difficult problem of initialization.

Regarding our approach of perceptual grouping, the initial picture of edge elements gives us the initialization required. The quality function involves also influences from the picture, such as grey levels or orientations, and from the curves with their cocircularity and their curvature. We show how the quality function can be written according to the active contour model and and how it provides us with the clear formalism and the independance we need between the curves and the picture.

3 A snakes-like quality function

In this section, we are going to define characteristics of curves in order to determine their quality. From these characteristics, we derive the expression of snake-like terms, used in the optimization process. We show eventually how these terms can be written in a recursive way for more efficient computation and a global optimization from a local level.

3.1 Qualifying the quality of a curve

Low level processings, such as edges segmentation or crest-lines segmentation, produce pictures containing flaws due to their sensitivity to noise or to local variation of the intensity function. The curves we want to extract from these images contain gaps, discontinuities and additional pixels introduced either by noise, weak strength of edges, occlusions or textures. Our goal is to extract curves as long and as smooth as possible.

Along the path of a curve, the solution we optimize has to match as many pixels of the original curve as possible while ignoring pixels of noise and filling gaps. This behavior can be reinforced if the tangents along the solution matche local orientations in the segmented image when available.

However the attraction imposed by pixels and local orientations is very important and has to be counter-balanced by shape constraints. Curvature is an elementary way to describe the shape of a curve. In order to obtain stable results, early experiments lead us to add co-circularity as a higher order derivative term for the description of the curve such as suggested by Zucker [15]. According to the importance we give to this factor, it's possible to preferentially extract loops or open curves.

As one can notice, these criteria are opposed. Pixels and orientations represent influences of the image on the curves we want to optimize (*external influences*) whereas curvature and co-circularity represent inner qualities of the curves (*internal influences*).

Regarding the model of snakes, the curvature and co-circularity quality functions represent the internal energy of a snake, whereas the grey-levels and orientation quality functions represent the attractive external energy of the picture. The internal energy of a snake is composed of a term of tension (length of the snake, term of first order) and a term of elasticity (curvature, term of second order). Regarding our problem, since we do not know any of the extremes of the curves, we are trying to optimize, the only information available is local to each point. Consequently, it is impossible to compute the length of the curves during the optimization process. Finally, our internal quality term is composed of a term of elasticity (curvature, term of second order) and a term of co-circularity (co-circularity, term of third order). However, instead of minimizing energy terms like in snakes models, our quality terms are optimized when reaching a maximum for the quality function

By optimizing our quality terms computed locally into a global process, we obtain a criterion whose value is high for long smooth or circular curves of high edge intensity with respect to local tangents on the image. The following section shows how each term can be written recursively for a local to global computation.

The complete quality function is defined as a linear combination of four quality terms and can be written, for a given pixel P:

$$\mathcal{F}(\,P\,) = \begin{cases} & \alpha\cdot\mathcal{C}(\,P\,)+\beta\cdot\mathcal{K}(\,P\,) \quad (\,internal \quad terms\,) \\ + & \gamma\cdot\mathcal{G}(\,P\,)+\delta\cdot\mathcal{O}(\,P\,) \quad (\,external \quad terms\,) \end{cases} \tag{1}$$

Where $(\,\alpha\,,\ \beta\,,\ \gamma\,,\ \delta\ \in\ [\,0\,,\,1\,]\,)$ are parameters rating the influence of the different terms. The importance and sensitivity of each parameter are discussed in section 5.

3.2 Regulation and Image Quality Functions

- **Global curvature and co-circularity functions:** *Internal terms*

  The quality factor of a curve arriving in a pixel j is defined as the sum of the local curvatures along the trace of the curve entering in and the curve exiting from j , with a factor $0\ \leq\ \rho\ \leq\ 1$ representing the attenuation of the quality with distance. If we write the relation as a bi-lateral function of the trace, with a trace coming in j $(\,\mathcal{C}_l(\,j\,)\,)$ and a trace going from j $(\,\mathcal{C}_r(\,j\,)\,)$, the quality becomes:

  $$\mathcal{C}(\,j\,) = (\,\mathcal{C}_r(\,j\,)+\mathcal{C}_l(\,j\,)\,)\cdot f_{j-1,j+1} \tag{2}$$

  with :

  $$\mathcal{C}_l(\,j\,) = \frac{1}{2}+\rho\cdot f_{j-2,j}+\rho^2\cdot f_{j-3,j-1}f_{j-2,j}+... \tag{3}$$

  where l references terms before j along the trace (j-1, j-2, ..) r references terms after j along the trace (j+1, j+2, ..) and $f_{j-1,j+1}$ represents the evalutation of a local curvature for the pixels $(\,j-1\,), j, (\,j+1\,)$,see Appendix A for more details about $f_{j-1,j+1}$
  Each term of this quality function is representative of a long distance measure of the quality of the curve. The factor ρ defines the manner in which long distance portions of curves influence the pixel j. In the same way, we can define the other functions as sums of local terms.
  Along the trace of the curve entering in the pixel j, with an attenuation factor $0\ \leq\ \omega\ \leq\ 1$, the quality function for co-circularity can be defined as :

  $$\mathcal{K}_l(\,j\,) = \kappa_{j-2,j-1,j+1}+\omega\cdot\kappa_{j-3,j-2,j}+\omega^2\cdot\kappa_{j-4,j-3,j-1}+... \tag{4}$$

  The global term is a sum of lateral contributions : $\mathcal{K}(\,j\,) = (\,\mathcal{K}_r(\,j\,)+\mathcal{K}_l(\,j\,)\,)$
  where $\kappa_{\mathrm{j-2,j-1,j+1}}$ is the evaluation of co-circularity for the pixels $(\,j\,-\,2\,), (\,j\,-\,1\,), j, (\,j\,+\,1\,)$. This term is also developped in Appendix A.

- **Grey levels and Orientation functions :** *External terms*

  Following the same formalism as the previous quality function, if σ_j represents the grey level of the pixel j of a trace, the grey level function for one of the lateral parts of the trace $(\,\mathcal{G}(\,j\,) = (\,\mathcal{G}_r(\,j\,)+\mathcal{G}_l(\,j\,)\,)\,)$ can be defined as :

  $$\mathcal{G}_l(\,j\,) = \sigma_{j-1}+\xi\cdot\sigma_{j-2}+\xi^2\cdot\sigma_{j-3}+... \qquad 0\ \leq\ \xi\ \leq\ 1. \tag{5}$$

Generally, low level processings can give us the gradient orientation Θ_T for a pixel P. These orientations can be used as well as grey levels to define an image orientation quality function :

$$\Phi_j = \exp^{\left(-Tan \left| \Theta_T - \widehat{(j-1).j.(j+1)} \right| \right)} \tag{6}$$

where $\widehat{(j-1).j.(j+1)}$ is the orientation of the connexion between the pixel j and its neighbors $j-1$ and $j+1$. This criterion is maximal when the two orientations are equal.
As the curve has to match the orientations in the segmented image as much as possible, we can define a global orientation function in the same way as the previous ones ($\mathcal{O}(j) = (\mathcal{O}_r(j) + \mathcal{O}_l(j))$) with :

$$\mathcal{O}_l(j) = \Phi_{j-1} + \lambda \cdot \Phi_{j-2} + \lambda^2 \cdot \Phi_{j-3} + ... \qquad 0 \leq \lambda \leq 1. \tag{7}$$

3.3 Recursive computation of the global quality criterion

The functions we have defined can be recursively computed by a progressive lengthening of the trace. We will first look at the 1-D case, followed by the 2-D case.

- **1-D case**

  We are interested here in writing recursively the quality functions defined by (3 (4)(5)(7) We first limit ourselves to parametized curves. We will see in a later subsection how to derive this calculation in the 2-D case. Deriving recursive expressions for these terms is important, as it allows computation of a global value for a curve using local operations exclusively. Each pixel is then considered as a processor, computing a new value locally and receiving global information from its neighbours.

  For example, the grey-levels quality function at length n is defined as the sum of the n first terms of $\mathcal{G}_l$ (resp. $\mathcal{G}_r$). We define with n the level of globality of our function (n represents the length of the portion of the considered trace).

$$\mathcal{G}_l{}^{(n)}(j) = \begin{cases} \sigma_{j-1} & \text{for} \quad n = 0 \\ \sigma_{j-1} + \xi \cdot \mathcal{G}_l{}^{(n-1)}(j-1) & \text{for} \quad n \neq 0 \end{cases} \tag{8}$$

  We define $\mathcal{G}_r$ in the same way.

- **2-D case**

  In the general case of two dimensions, we must take into account all the possible traces at a given pixel P. We call $V(P)$ the neighboring system around P (see Figure 2) and $N = Card(V(P))$ is the size of $V(P)$.
  This 2-D case differs from the previous one in that numerous possible traces can "cross" a single pixel. In order to optimize globally the quality function over the image, we have to compute, for each pixel P, the best traces crossing it. Therefore, for each trace entering in P, we look for another entering trace such that the local quality function is maximal for this couple of curves. This leads to select N couples of pixels from among $N(N-1)$ possible ones. For that calculation, we also need the information of exiting traces (a trace crosses a pixel). We construct a binary connection matrix, defining the N traces crossing each pixel. In 2-D, we obtain for each quality function in P in the direction i (with $Q_i = Neighbour(i,P)$, pixel in $V(P)$ corresponding to i) :

- Regulation Quality Terms

$$
\begin{array}{ll}
Initialization \;\; (n=0): & \mathcal{C}_i^{(0)}(P) = 1 \\
 & \mathcal{K}_i^{(0)}(P) = \kappa_{e',\imath,o} \\
For \;\; n \;\neq\; 0 \;\; : & \\
 & \mathcal{C}_i^{(n)}(P) = 1 + \rho \cdot f_{e',\imath}\mathcal{C}_{e'}^{(n-1)}(Q_i) \\
 & \mathcal{K}_i^{(n)}(P) = \kappa_{e',\imath,o} + \omega \cdot \mathcal{K}_{e'}^{(n-1)}(Q_i)
\end{array}
\tag{9}
$$

- Image Quality Terms

$$
\begin{array}{ll}
Initialization \;\; (n=0): & \mathcal{G}_i^{(0)}(P) = \sigma_{Q_i} \\
 & \mathcal{O}_i^{(0)}(P) = \Phi_{Q_i} \\
For \;\; n \;\neq\; 0 \;\; : & \\
 & \mathcal{G}_i^{(n)}(P) = \sigma_{Q_i} + \xi \cdot \mathcal{G}_{e'}^{(n-1)}(Q_i) \\
 & \mathcal{O}_i^{(n)}(P) = \Phi_{Q_i} + \lambda \cdot \mathcal{O}_{e'}^{(n-1)}(Q_i)
\end{array}
\tag{10}
$$

with :

$$e' \;=\; entering(\,\bar{\imath},\; Q_i\,) \quad , o \;=\; exiting(\,\imath,\; P\,)$$

(e' and $\bar{\imath}$ are seen from Q_i). The *entering* (resp. *exiting*) function correlates a given direction o of a pixel P with the entering direction i such as the quality function is maximal along the path ($i \rightarrow P \rightarrow o$).

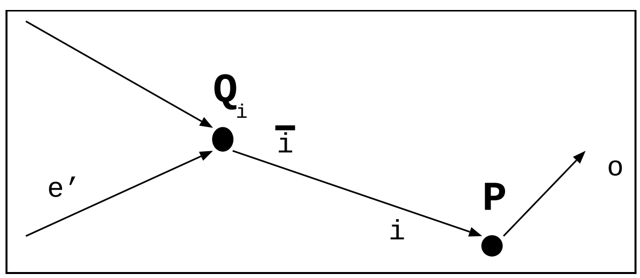

Fig. 1. Notations used for a connection

In this sense, we optimize a two-sided quality function made of two lateral entering contributions. Here also, the global terms are defined as sums of bilateral functions, but they take into account all the possible connections between entering and exiting paths on a given pixel.

4 Optimization of the quality function

We propose here a dynamic method to optimize the quality function iteratively, from a local to a global level.This method is related to Dynamic Programming [2] as described originally by Sha'ashua [11]. However this method uses a different quality function. The algorithm for grouping and for extracting the salient solutions differ completely from the original method.

As said in the previous section, to group pixels together we are represent them surrounded by a set of neighbours. We use a neighbouring system with 16 pixels, which allows a wider range of values for local curvatures than the usual 8-connectivity (see section 5 about the choice of neighbouring system).

For each pixel, we define a connection by a pair of neighbouring pixels representing the directions of arrival and departure for a possible curve crossing the pixel. We select the pair of neighbours giving the best value for the quality function. The recursive expression of the quality functions allows us to compute their values with a local part (defined by the local characteristics of the connection) and a global contribution provided by each neighbour. For each iteration, this global contribution takes into account more distant pixels. This mechanism increases the global level for the quality functions along the iterations.

Once the optimization has been performed, the curves are extracted by following the connections from one pixel to another until the curve crosses its own trace, reaches a boundary of the image or comes to a dead end (for example, when the curve has traced a certain distance without encountering pixels from the segmented image). The number of possible curves on the image is reduced to a single optimised curve for each possible starting point. However it is still high enough to make a selection process remain necessary.

4.1 Computation of the connections

More formally, the connections are computed in two steps: 'inputs toward outputs' then 'outputs toward inputs'. Basically, for each pixel and each of his neighbours (called *input*) we first select the neighbour giving the best quality. The neighbour selected is called an *output* for the connection. There can be many inputs associated to the same output.

- **Connecting inputs toward outputs**
  For each pixel P and each input Q_i, we look for an exiting pixel Q_o which maximizes the quality function at step n in (P): $\mathcal{F}^{(n)}_{Q_i,Q_o}(P)$

  Let $C^{(n)}(P)$ be the connection matrix for a given pixel P at step n. Its elements are of the following form:

$$C^{(n)}_{Q_i,Q_o}(P) = \begin{cases} 1 \quad if \quad \mathcal{F}^{(n)}_{Q_i,Q_o}(P) = \max_{Q_o{}' \in V(P) \setminus \{Q\}} \mathcal{F}^{(n)}_{Q_i,Q_o{}'}(P) \\ \\ 0 \quad ifnot \end{cases} \tag{11}$$

  One can notice that the connection of inputs toward outputs is not symmetrical: for an entering direction i and a pixel P, $exiting(i, P)$ is the only exiting direction, the quality function of which is maximum but the opposite is false.

Algorithm 1: Algorithm of connecting and updating of quality functions.

begin
% At the iteration n :
% *Connecting Inputs toward Outputs*
For each *Pixel* P **do**
For each *entry* i **do**
We seek an optimal *output* o for $\mathcal{F}^{(n)}_{i,o}(\ P\ )$
[1] ... with the constraint ($Curvature_{i,o} > 0.05$)
$exiting(\ i, P\ ) \leftarrow o$
endfor
endfor
%
% *Updating of quality functions and Connecting Outputs toward Inputs*
For each *Pixel* P **do**
For each *input* i **do**
[2] $o \leftarrow exiting(\ i, P\ )$
$Q \leftarrow Prec(\ i, P\ )$ % Q is the pixel in $V(\ P\ )$ corresponding to i
[3] We seek an *input* e in Q such as : ($\mathcal{F}^{(n)}_{e,\bar{\imath}}(\ Q\ ) = \mathcal{F}^{(n)}_{max}(\ Q\ )$) and ($exiting(\ e, Q\ ) = \bar{\imath}$)
if e *exist* **then** $l \leftarrow e$ **else**
[4] $l \leftarrow exiting(\ \bar{\imath}, Q\ )$
endif
[5] **if** ($\mathcal{F}^{(n)}_{\bar{\imath},l}(\ Q\ ) < \mathcal{F}^{(n)}_{\bar{\imath},exiting(\ \bar{\imath},Q\ )}(\ Q\ )$) **then**
$l \leftarrow exiting(\ \bar{\imath}, Q\ )$
endif
[6] $entering(\ i, P\ ) \leftarrow l$
$\mathcal{G}(\ P\ )[i] \leftarrow \sigma(\ P\ )[i] + \xi \cdot \mathcal{G}(\ Q\ )[l]$
$\mathcal{C}(\ P\ )[i] \leftarrow \frac{1}{2} + \rho \cdot \mathcal{C}(\ Q\ )[l] \cdot f_{l,\bar{\imath}}$
$\mathcal{K}(\ P\ )[i] \leftarrow K_{(\ o\ ),(\ i\ )(\ l\ )} + \omega \cdot \mathcal{K}(\ Q\ )[l]$
$\mathcal{O}(\ P\ )[i] \leftarrow \Phi(\ i, P\ ) + \lambda \cdot \mathcal{O}(\ Q\ )[l]$
endfor
endfor
end

[1]

1. Curvature constraint to avoid narrow connexions.
2. We look first for the optimal input. If no input is available, we connect with the output.
3. For Q, the output we try to connect is $\bar{\imath}$, l is the input in Q we are looking for.
4. No input is connected to $\bar{\imath}$: We consider the reverse connection, $\bar{\imath}$ is viewed as an input to connect .
5. We keep the best connection for $\bar{\imath}$ in Q between l ("ouput toward input") and $exiting(\ \bar{\imath}, Q\ )$ ("input toward output").
6. Updating of the quality function terms in P for the direction i.

– **Connecting outputs toward inputs**

To be sure to select the best connection, we associate, when possible, each output with the input giving the best quality. Input traces come from a long distance as output traces are influenced by the local noise. Thus, connecting outputs with inputs results in less sensitivity to noise than would exist by connecting inputs to outputs only.

We are going to define the *entering* function ($i = entering(\ o,\ P\ )$) which gives, for any exiting direction o of a pixel P, a corresponding entering direction i. There are two possible cases: one or several entering directions may exist, or no corresponding input exists.

<u>Case of no input:</u>
In this case, a given output Q_o has no corresponding input. We consider Q_o as a possible input and we keep the connection already defined between inputs towards outputs. We define the function *entering* as:

$$entering\,(o, P) = y \quad with \quad C^{(n)}_{Y,Q_o}(P) = 1$$

<u>Case of multiple inputs:</u>
In the case of multiple inputs, we have to make a choice between L possible inputs: we define the *entering* function, such that the quality function in P with this output o is maximal for the input i_k among the L possible inputs.

More formally, we have: $Q_o = Neighbour\,(\ o,\ P\ )$
let: $E\,(Q_o) = \{(Q_o, Q_{i_0}), (Q_o, Q_{i_1}), ..., (Q_o, Q_{i_L})\}$

We keep the pair (Q_o, Q_{i_k}) such that $\mathcal{F}^{(n)}_{Q_{i_k},Q_o}(\ P\ )$ is maximum.

We can now define the input i_k such that Q_{i_k} is the optimal neighbour entering in P along direction i_k, ($Q_{i_k} = Neighbour\,(\ i_k,\ P\ )$). Between this connection "output toward input" (Q_{i_k}, Q_o) and the previous connection "input toward output" $(Q_{i_k}, exiting\,(i_k, P))$, we keep the connection that gives the best quality function.

Let: $q = exiting\,(i_k, P)$
We define eventually :

$$entering\,(o, P) = \begin{cases} i_l \ \text{ if } \ \mathcal{F}^{(n)}_{Q_{i_l},Q_o}(\ P\ ) > \mathcal{F}^{(n)}_{Q,Q_o}(\ P\ ) \\ \\ q \quad \text{if not} \end{cases} \tag{12}$$

The complete optimization algorithm is described in Algorithm 1

4.2 Global Optimization and Selection of the best paths

Up to now, we have presented a "snake like" recursive quality function and a local optimization algorithm using connection matrices. We come up with a single optimized curve for each possible starting point on the image. It is now necessary to differenciate these curves from each other and select the most salient of them.

Along the iterations, the connections are organized locally, allowing the emergence of some salient paths throughout the network. In order to select the most salients path among them, we explore these paths and evaluate their global quality. For a given pixel, we follow the connections from one pixel to another in the direction maximizing the local quality function (this will avoid the exploration of directions which do not seem promising) until there is no further connection to follow. We end up with a chain of pixels representing the best path from the original point according to the optimization.

Algorithm 2: Algorithm of optimization and selection.

begin
  Initialization of quality functions and connexions at step 0 ($n = 0$)
  %
  % **Iterations**
  For each *Pixel* **do**
    For each *entry* **do**
      Local optimization of the quality function : computation of the connexions
      Update of the quality function
    endfor
  endfor
  %
  % **Following and selection the best paths**
  For each *Pixel* **do**
    Follow the chain from current pixel
    Compute quality of the chain
    Update buffer
  endfor
  For each *Pixel* $P \notin$ *Chains already selected* **do**
    if *(Buffer(P) > Threshold)* **then** Select Current Chain
  endfor
end

We define the global quality of a chain as the sum of local qualities for each of its points. As it is possible to follow a chain starting with every possible pixel of the image, this definition for the global quality gives us a group of chains with equivalent qualities for each salient curve of the image. It is possible to reduce the number of chains in each groups by weighting their quality with an additional factor. If we consider the amount of pixels from the original image encountered by the chain, in regard to the amount of pixels followed by filling gaps in this image, we come up with a factor that can better differenciate the groupings. We define eventually the function which associates each pixel to the quality of the chain starting with it. A simple thresholding of this function allows us to automatically extract the best groupings.

Finally, we come up with a two stage algorithm, wich first optimizes, then extracts global information from all the connections matrices (algorithm 2)

5 Convergence and Implementation

5.1 Convergence

In the 2-D case, the problem of convergence becomes complex because connections change as the iterations increase and the series calculated here are non-monotonic. We verify experimentally on several images that the convergence of the algorithm is well maintained for various values of the parameters. Therefore, we notice that for very noisy input images, convergence sometimes becomes difficult ; the network can oscillate between several different salient solutions.

Along the iterations, isolated pixels of impulse noise see their importance decrease in regard to pixels included in large structures. Pixels within gaps see their quality increase by influence of the neighbouring structures.

Our quality function can be controlled by two sets of parameters.

- (α , β , γ , δ $\in$ [0, 1])
  These parameters represent the importance given to each term in the quality function. They are respectively related to the influence of curvature, co-circularity, grey levels and orientation terms on the shapes of the selected curves. For instance, for a high value of γ, the curves selected will tend to be attracted more by pixels. A high value of β will increase the importance of co-circularity and give a better quality to loops instead of open curves. It can take a certain number of trials to optimize exactly the class of curves expected, but once the correct settings are found, the detection remains significantly good for different images . As an example of the stability of our method, all the results in the following section have been computed with the same set of values.
- (ρ , ω , ξ , λ , $\in$ [0, 1])
  They represent the influence of distant contribution for each term of the quality function. A value of 0 will reduce the corresponding term to its local value only.

During the optimization process, the number of iterations is related to the distance between contour elements we want to connect. The whole algorithm has a behaviour of diffusion. This can be represented as if the influence of each pixel spreads in every direction one pixel further with each iteration. In order to fill gaps along the curves of the original image, the length of the wider gap gives us the minimum of iterations for the optimization process.

As it is an iterative process, there is no limit to the number of iterations allowed. We must remember that the optimized curves tend to be smoothed as they receive more global contributions. Thus, a high number of iterations means a loss in the precision of the selected curves. It can be interesting to select the best groupings at various levels of precision as they represent increasingly more global results.

5.2 Implementation

The 8-connectivity does not supply enough sampling angles to allow the precise computation of the curvature of a path to take place. We use a system of 16 neighbours : 8 arising from the 8-connectivity, and 8 neighbours defined by the moves of a knight in a chess game (see Figure 2). This neighbourhood offers an intermediate structure between the small 8-neighbourhood and larger

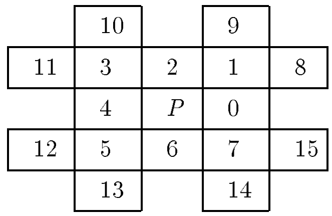

Fig. 2. *Neighboring system*

ones and it does not provide a high level of algorithmic complexity (dimension of $F_{i,o} = 16 \times 16$). Experiments have been carried out with a larger number of neighbours (32) without improved results (but an increased algorithm complexity). Results have been computed on a SUN Sparcstation LX.

6 Results

All the results in this section have been computed with the same parameters. One can notice the quality of the groupings after automatical selection for different situations (circles, ellipses and thin nets). The good quality of thses results obtained with the same parameters for very different classes of images shows clearly the robustness of our method. However, we show how it is possible to adapt parameters to the class of curves and get better results in fig 13.

6.1 Synthetic Images

We have tested our method on Synthetic images in two situations regarding the class of noise introduced in the images:white noise and gaussian noise (in the grey level function).

In the case of white noise, the images used represent an ellipse where %40 of the pixels have been removed by white noise (figs 3 and 4). %10 and %20 of pixels of white noise have then been added to this image. The ellipsoid shape is still recovered even with high level of noise.

In the second situation, the images represent an ellipse (fig 5) and a circle (fig 6) where gaussian noise has been introduced. Edges extraction in this case can't be performed efficiently at a low level but circular shapes are recovered after applying the perceptual grouping. For each image, we show the original image, the image used after edge extraction and the grouping that gave the higher quality.

6.2 Real Images

Grouping has been tested on both satellite and medical images. The segmented images in fig 8, fig 10 and fig 12 show the result of thin network extraction [8] . For each picture, we present the main groupings proposed by the selection algorithm (about 20 iterations were required for smooth and significant groupings on figs 8 and 11). It's important to remember that the selection algorithm extracts salient solutions according to their quality. This explains the missing groupings one can notice on the results images. It's always possible to obtain more groupings with a lower selection threshold.

These results show how it is possible to extract the main groupings for various kinds of curves with the same parameters for the quality function. Further tuning of these parameters allow us to get results more appropriate to the scene (for example, increasing the importance of curvature and reducing the importance of grey levels gives better shapes for fig 13).

The extraction of main groupings automatically with no prior knowledge about the shapes produces a first description of the scene and a good initialization for further higher level processings such as model based shape recognition or active contours optimization.

7 Conclusion

We presented the different optimization techniques developped for Perceptual Organization of thin networks and the quality functions used for this application.

We proposed improvements to the 'local to global' optimization scheme proposed by A. Sha'ashua and S. Ullman and we combined this scheme to models of snakes in order to obtain a better solution for Perceptual Grouping. We also proposed a novel iterative scheme for optimizing the quality functions using a different algorithm and another kind of quality functions. We then described an algorithm which extracts principal groupings automatically.

The algorithm proposed by Sha'ashua and Ullman has been changed in order to use more global information from the image. Our method can deal with dot images instead of segment images, which are obtained when no orientation from low level processing is available. It can extract a broader diversity of shapes, especially with the additional terms included in the quality function. The high quality of the results shows how the method is more robust to noise and how it can be easily applied to real situations such as roads or blood vessels detection.

Possible applications range from closing edges to the initialization of active contours or the extraction of unknown shapes in very noisy images. In the future, we plan to focus on more complex groupings and adapt this method to the extraction of 2D and 3D curves.

A Appendix

A.1 Definition of the elementary quality functions

We define here curvature and co-circularity terms that are used to define the global quality function of regulation.

- **Curvature**

  The total curvature of a trace (γ) gives us useful local curvature terms for the quality criterion.

$$C \;=\; \exp\left(-\int_{\gamma} \left(\frac{d\theta}{ds}\right)^2 ds \right)$$

or, in discrete terms after normalization :

$$C = \prod_{j=2}^{M-1} f_{j-1,j+1}$$

where M is the number of pixels of the trace. $f_{j-1,j+1}$ represents a measure of curvature between three consecutive pixels along the trace (seen as a parametrized curve).

$$f_{j-1,j+1} = \exp\left(- \frac{2\ \theta\ \tan\left(\frac{\theta}{2}\right)}{\Delta S} \right) \tag{13}$$

where θ represents the angle between $j-1$ and $j+1$ through j :

$$\theta = 2\pi - \widehat{j-1, j, j+1}$$

ΔS is the distance between the two pixels $j-1$ et $j+1$ (by j). Thus, we have $0 \leq f_{j-1,j+1} \leq 1$, and $f_{j-1,j+1} = 0$ if $\theta = \pi$, $f_{j-1,j+1} = 1$ if $\theta = 0$.

– **Co-circularity**
We also define a criterion of local co-circularity based on the second derivative of the orientation in regard to the curvilinear coordinates $\left(d^2\theta/ds^2 \right)$ of the curve. Due to numerical approximation, we chose to differentiate the local curvature terms ($f_{i,j}$) ; we obtain the 1-D parametric term defined as :

$$\kappa_{j-2,j-1,j+1} = \frac{\left| \operatorname{sgn}(\theta_{j-1})\ f_{j-2,j} + \operatorname{sgn}(\theta_j)\ f_{j-1,j+1} \right|}{2\ \Delta s} \tag{14}$$

where $j-2, \cdots, j+1$ represent four consecutive pixels along a curve and Δs represents the distance along the curve between $j-2$ and $j+1$.

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2 Figures

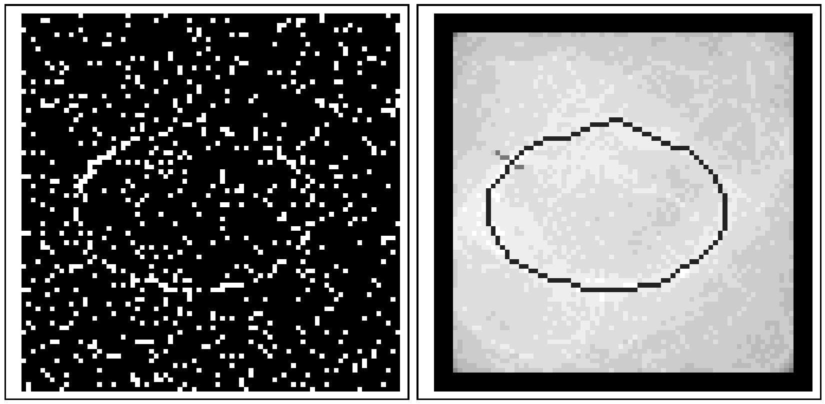

Fig. 3. Ellipse 80x80 with %20 of noise - 25 iterations (26 sec / iteration)

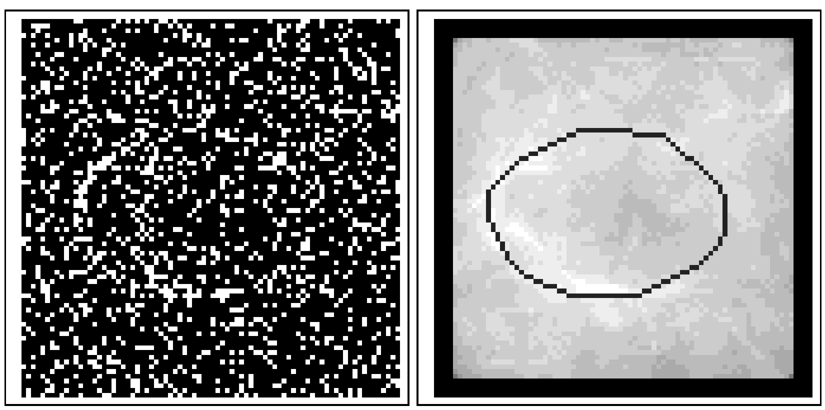

Fig. 4. Ellipse 80x80 with %20 of noise - 25 iterations (26 sec / iteration)

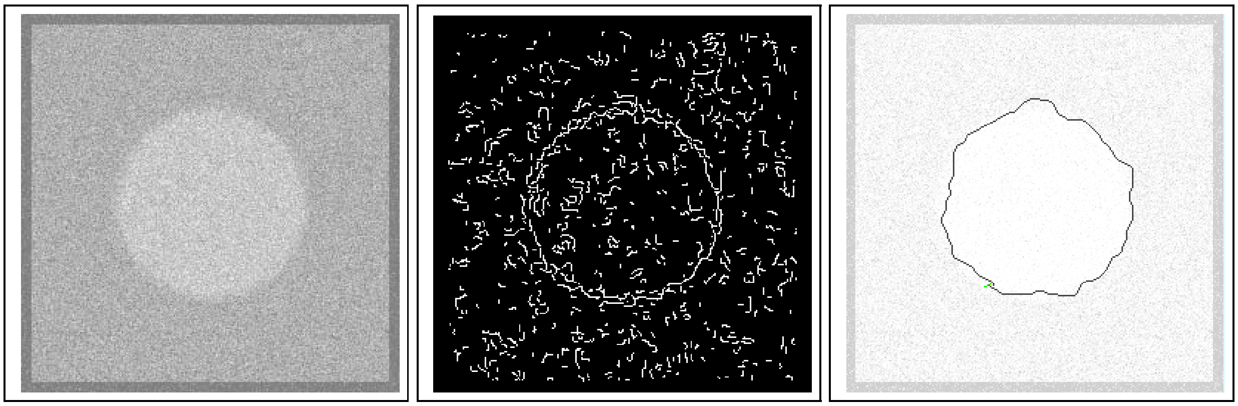

Fig. 5. Circle 256x256 with gaussian noise - 10 iterations (5 mins / iteration)

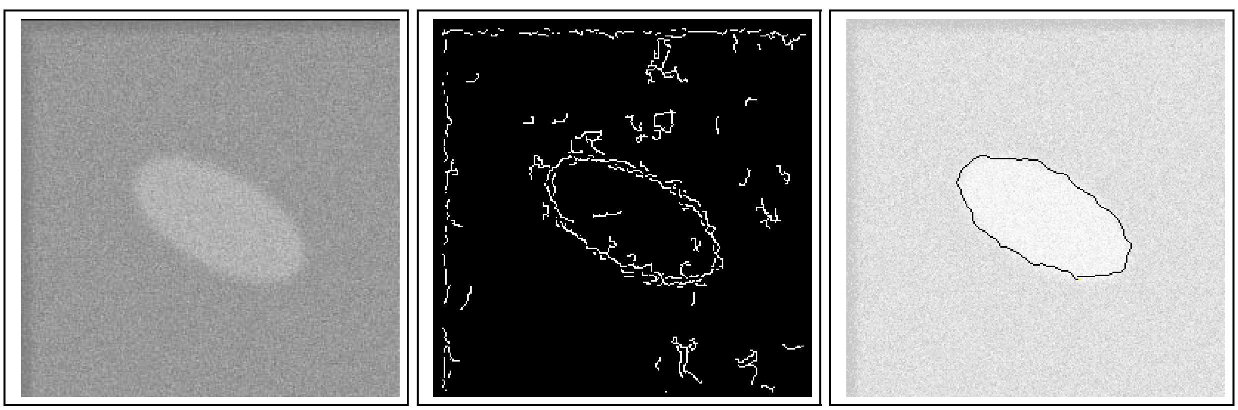

Fig. 6. Ellipse 256x256 with gaussian noise - 10 iterations (5 mins / iteration)

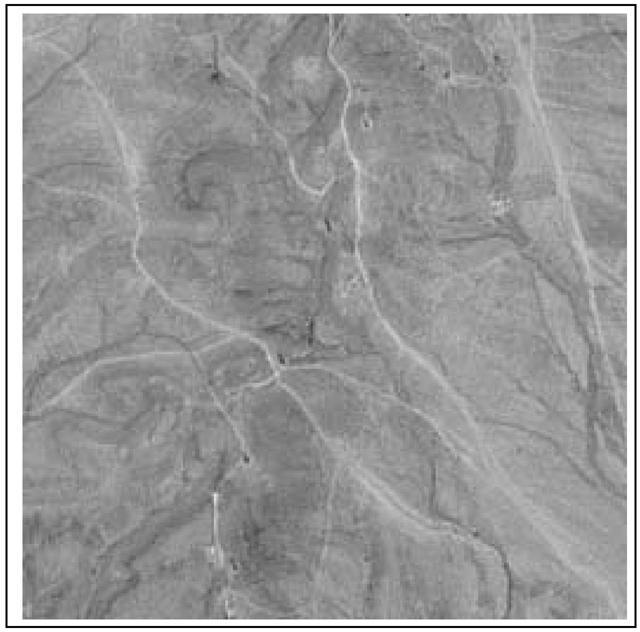

Fig. 7. Satellite Picture 256x256 - Original image

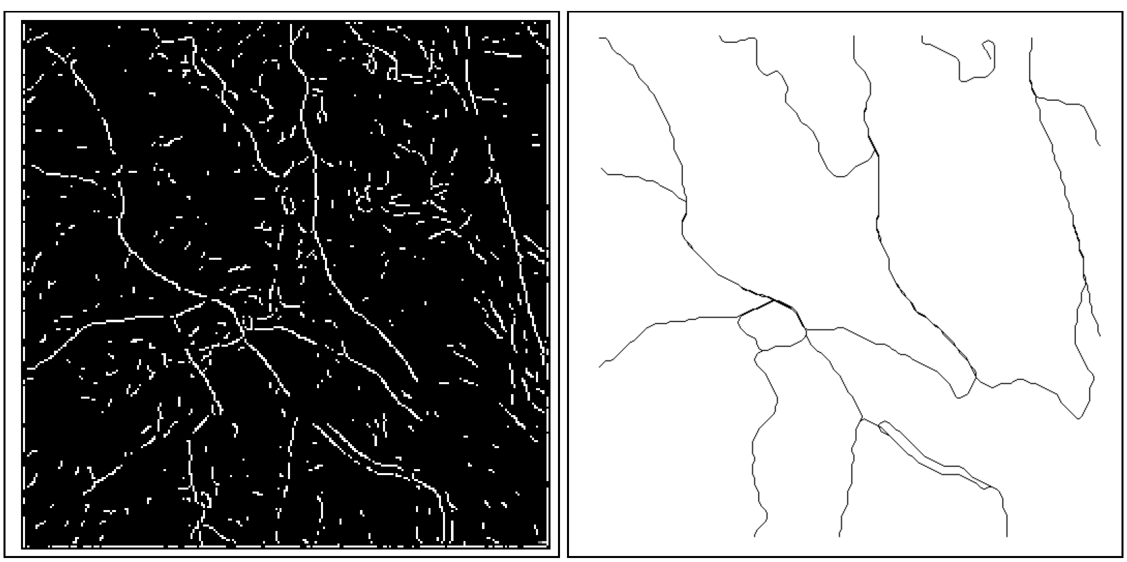

Fig. 8. Satellite Picture 256x256 - Segmented image and Final selection of 14 main groupings

Fig. 9. Satellite Picture 256x256 - Original image

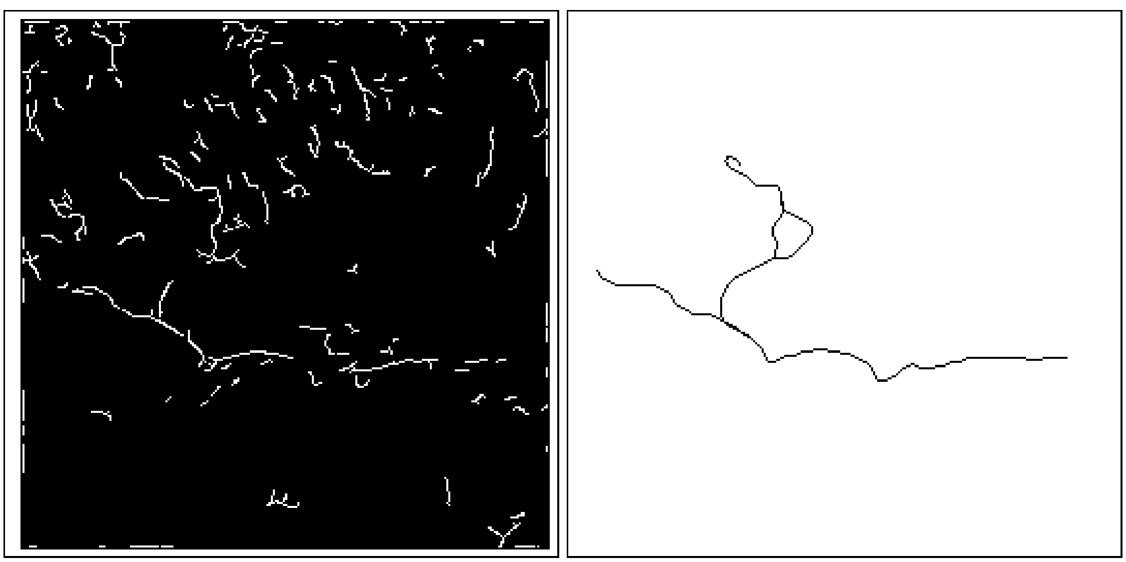

Fig. 10. Satellite Picture 256x256 - Segmented image and Final selection of the 3 main groupings

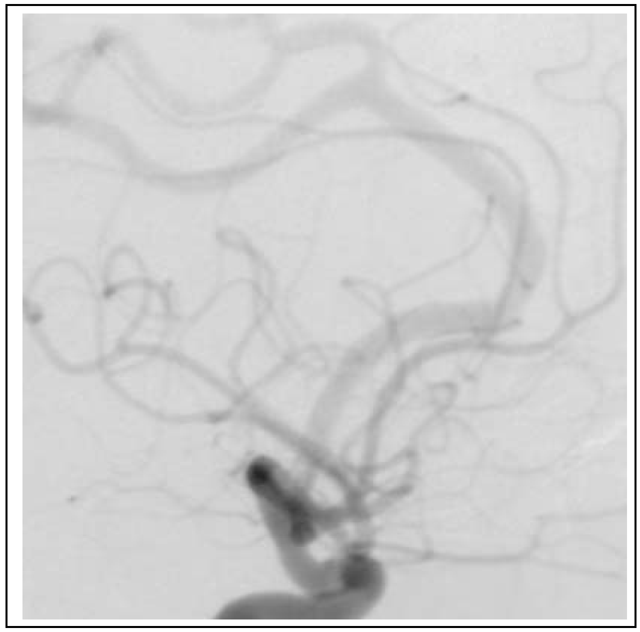

Fig. 11. Medical Images 400x400 - Original image

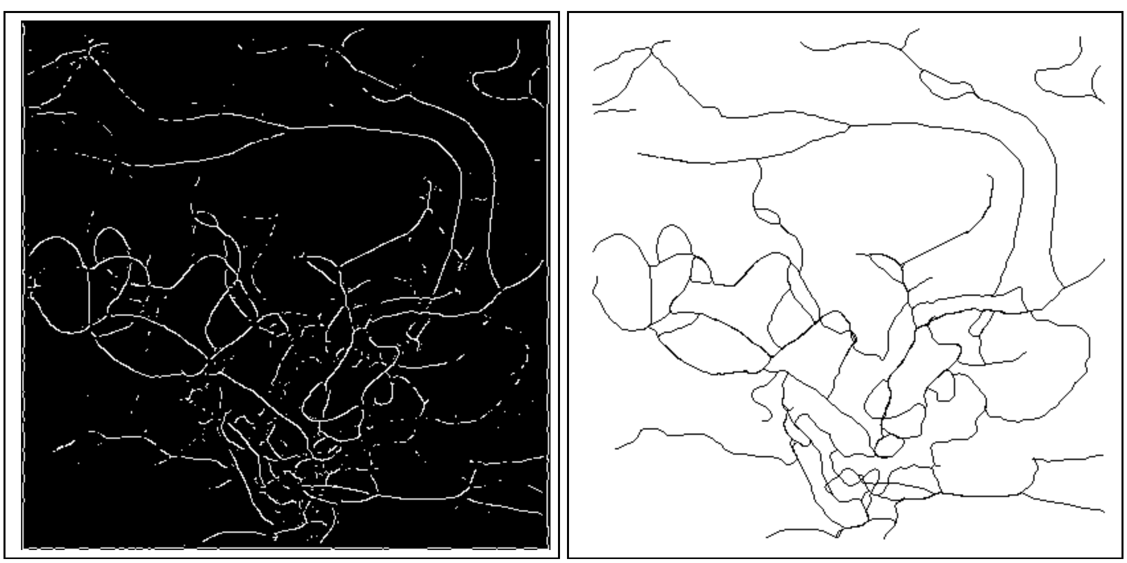

Fig. 12. Medical Images 400x400 - Segmented image and Selection of the 50 main groupings

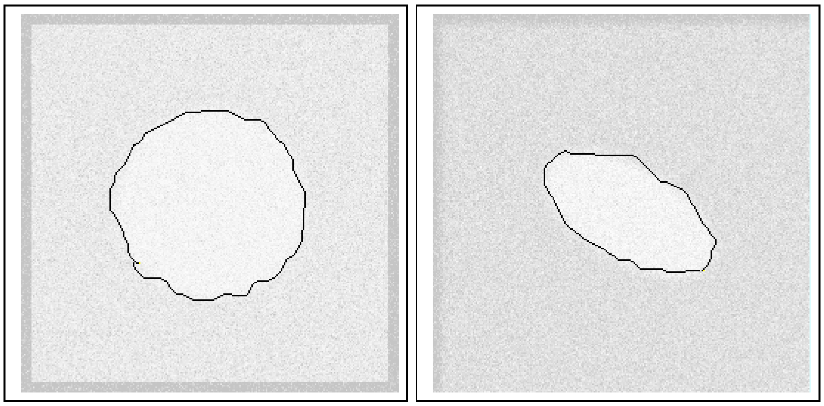

Fig. 13. Circle and Ellipse 256x256 with gaussian noise - 15 iterations (5 mins / iteration)